ALGEBRAIC THINKING IN PRIMARY SCHOOLS: AN INSTRUMENT VALIDATION USING PLS-SEM

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Abstrak: Sejak tahun 2000, bidang pemikiran algebra dalam kalangan murid sekolah rendah semakin mendapat perhatian daripada penyelidik pendidikan matematik. Pemikiran algebra merangkumi tiga topik yang utama; iaitu aritmetik umum, pemodelan dan fungsi. Dengan mengambil kira evolusi ini, instrumen untuk mengukur pemikiran algebra murid sekolah rendah adalah amat penting. Oleh itu, kajian ini bertujuan mentaksir ujian diagnostik pemikiran algebra dari aspek kesahan konstruk dengan menggunakan kaedah Model Separa Kuasa Dua Terkecil- Pemodelan Persamaan Struktural (PLS-SEM). Ujian diagnostik ini ditadbir kepada 539 murid Tahun Lima di suatu daerah di Melaka. Sampel kajian terdiri daripada 51% murid lelaki dan 49% murid perempuan. Model formatif-formatif telah digunakan. Ketiga-tiga konstruk pemikiran algebra telah ditaksir dan ditentukan kesahannya melalui pendekatan dua-peringkat. Penilaian model komponen lower-order mengesahkan bahawa tiada isu multikolineariti antara indikator. Penilaian model komponen higher-order mengesahkan bahawa tiada isu multikolineariti antara konstruk. Dapatan kajian ini menunjukkan ujian diagnostik ini boleh digunakan untuk mentaksir tahap pemikiran algebra murid Tahun Lima. Ujian ini juga boleh digunakan sebagai kajian masa depan yang bertujuan mengukur pemikiran algebra murid Tahun Lima. Selain itu, ujian ini juga mengenalpasti konstruk yang berkaitan dengan kandungan pemikiran algebra yang boleh digunakan oleh pendidik dalam kelas pengajaran.

Kata Kunci: pemikiran algebra awal, model pengukuran formatif, permodelan persamaan struktural

PENGENALAN

Lately, fostering algebraic thinking is gaining attention in primary schools around the world. Ample studies have been conducted to investigate primary school pupils' algebraic thinking in various perspectives. The perspectives include generalisation of arithmetic, ability to work with patterns, properties of operations, zeroes and ones, understanding of equal sign, and working with unknowns (Carpenter, Levi, Berman, & Pligge, 2005; Knuth, Stephens, McNeil, & Alibali, 2006; McNeil & Alibali, 2005; Warren, Cooper, & Lamb, 2006). The majority of these studies were qualitative while a few were quantitative. However, there are limited validated instruments available to measure all aspects of primary school pupils' algebraic thinking (McNeil & Alibali, 2005; Rittle-Johnson, Matthews, Taylor, & McEldoon, 2011; Ralston, 2013).

Li, Ding, Capraro, and Capraro (2008) compared sixth grade U.S. and Chinese pupils' equivalent understanding by using a scale comprising of four items. Similarly, Rittle-Johnson, Matthews, Taylor, and McEldoon (2011) developed and measured primary school pupils' algebraic thinking from the perspective of equivalence. Their sample involved 175 subjects from second grade to sixth grade who completed a written assessment involving items on an understanding of mathematical equivalence. However, considering other aspects of algebraic thinking such as functional thinking and ability to work with variables, measures on equivalence alone are insufficient as an algebraic thinking measurement tool for primary school pupils.

As such, Ralston (2013) has developed a measurement tool to measure algebraic thinking of second grade to fifth grade primary school students. This instrument comprises all the different perspectives of early algebraic thinking in the literature. While other measurement instruments in the literature are only limited to one strand of algebraic thinking (i.e., equivalence), this instrument included three main strands of algebraic thinking -- modelling, generalised arithmetic, and function -- as proposed by Kaput (2008).

Firstly, modelling comprises the students' ability in solving open number sentences, equality, and working with unknowns or more commonly known as variables. Ability to work with variables will provoke students' understanding properties of operations and capability to manipulate the operations. For instance, multiplication is actually repeated addition, addition and multiplication are commutative, and addition is the inverse of subtraction. Equal sign plays an important role in investigating students' understanding of equality. Understanding of equality helps students to simplify and solve complex problems. For example, when given that 55 + 37 = 54 + __; understanding of equivalence will enable a student to solve it using compensation strategy (i.e., add 1 to 37) instead of solving the problem left to right (Jacobs, Franke, Carpenter, Levi, & Battey, 2007). Working with unknown does not really require an understanding of the true meaning of variable (Blanton et al., 2015; Ralston, 2013). At this stage, it is just focused on students' common sense that a letter represents a number and they then work accordingly to solve the problem.

Secondly, generalised arithmetic is about working with generalisation and efficient numerical manipulation. Understanding the properties of zero will enable a student to generalise that multiplication of zero by any number would result in zero. Ability to generalise would mean knowing that addition of two odd numbers will produce an even number. Efficient numerical manipulation refers to the capability to apply properties of operations appropriately (Ralston, 2013). When given a problem such as $(9 \times 57) + 57$; students should be able to see it as adding another 57 to (9×57) means 10×57 rather than working from left to right. Thus, students should easily state the answer as 570. The ultimate purpose of this problem is not to get the correct answer but to investigate students' capability of manipulating the problem efficiently which later will be an essential skill required in formal algebra.

Lastly, function involves the ability to understand and work with figural and numerical patterns (Lannin, Barker, & Townsend, 2006). Students should be able to make relations between context, picture, table, and function rule. The patterns can be a sequence of numbers or sequence of figures. When students are able to sense the relationship between first and subsequent terms, they will perform near generalisation successfully. In both figural and numerical types, students are required to find a general 'rule' skill. This skill acquisition will enable them to perform far generalisation. Far generalisation is whereby they can make use of the 'rule' to find any arbitrary terms without calculating all the terms in the sequence.

The majority of studies on primary school pupils' algebraic thinking are qualitative in nature. To date, only one instrument is available for measuring all perspectives of algebraic thinking (Ralston, 2013). The present study aimed at validating Ralston's (2013) instrument on primary school pupils' algebraic thinking using PLS-SEM. But first, to put the research in perspective, the following sections discuss differences between the reflective and formative measurement model, and past studies carried out on primary school pupils' algebraic thinking. Partial Least Square (PLS) analysis, measurement model assessment, and two-stage modelling approach were used in the present study. Lastly, the findings were discussed and concluded.

Reflective versus formative measurement model

An unobservable attribute that cannot be measured directly is known as a construct. In structural equation modelling (SEM), it is also known as a latent variable. Commonly, latent variables are measured by items or indicators in instruments (Diamantopoulos & Siguaw, 2006). The underlying indicators of a latent variable can be reflective or formative (Chin, 1998). There is no clear-cut guideline on how to decide whether the indicators should be reflective or formative (Hair, Hult, Ringle, & Sarstedt, 2014). Researchers should make the decision on the indicators involved and how it should be represented in the measurement model. Reflective measurement model involves causal relationship from construct to indicators. The indicators measure the same construct and therefore should be highly correlated and interchangeable. The formative measurement model involves causal relationship from indicators to the construct. The indicators measure different aspects of the underlying construct. Hence, the indicators should not be highly correlated because they are independent causes.

Past studies on algebraic thinking in primary schools

Algebra has been portrayed as school mathematics' most crucial "gatekeeper" (Cai & Moyer, 2008). Therefore, students' misconception and inadequate preparedness in learning algebra have been major concerns of mathematics researchers in recent years. To overcome this problem, researchers have proposed to develop algebraic thinking starting from primary school level which can help to improve children's understanding of algebra in later years of education. As such, many studies have been conducted to investigate primary school pupils' algebraic thinking. As discussed earlier, algebraic thinking can be viewed from various perspectives. Equivalence, generalised arithmetic, functional thinking, and working with variables are major strands of algebraic thinking (Blanton et al., 2015). The following sections discuss a few related studies on these big strands.

Understanding of algebra begins with a strong foundation in equality concept, especially the conceptual meaning of the equal sign (Carpenter et al., 2005; Jacobs et al., 2007; Knuth et al., 2006). Equal sign is usually introduced to children at very early age and unfortunately, little emphasis is given to it in later years of education. Equal sign plays an important role in equality to signify both sides are equal. As such, McNeil and Alibali (2005) addressed equation as "any mathematical statement that uses the equal sign to indicate that two mathematical expressions are (or are defined to be) equivalent" (p. 883). Numerous studies have revealed primary school pupils have a huge misconception about the equal sign (Carpenter et al., 2005; Matthews, Rittle-Johnson, McEldoon, & Taylor, 2012). Because of this misconception, they have difficulties in solving equations especially operations on both sides of the equal sign (i.e., 6 + 2 = 4 + __). With regard to this, Carpenter et al. (2005) have proposed to provoke children's relational thinking while teaching traditional arithmetic rather than focusing on computation which only leads to the correct answer.

To test the contribution of equal sign knowledge in solving equalities, McNeil and Alibali (2005) investigated if the equal sign misconception associated with the difficulties in equations. The authors addressed three main operational patterns on how children infer equal sign. In the equation such as 3 + 4 + 5 + 6 = __, children tend to infer equal sign as i) equation-solving strategy, ii) "operations = answer", or iii) the total. Equation-solving strategy refers to students' attempt to compute all numbers based on all operations given. Secondly, "operations = answer" is when students are very used to perceptual patterns to the equations structure whereby all operations should be on the left side of the equal sign followed by answer on the right side. Thirdly, equal sign means "the total". Students view equal sign as meaning they should provide "the total". Thus, they just add all the numbers regardless of the equal sign position.

Based on the three items provided to 91 children in the age range of 7-11 years, the results were analysed based on the students' mindset to the patterns of operations and the effect of lessons conducted on equations. Results showed operational patterns adherence negatively correlated with learning; whereby children were most likely to develop correct equation solving strategies after a brief lesson if they are not adhering to operational patterns. These findings are supported by Carpenter et al. (2005) who found that focusing on relational thinking rather than mere computation algorithms to find answers can provide enhanced arithmetic learning and also generate more consistent types of knowledge to support formal algebra learning in future. In other words, the traditional arithmetic teaching and learning in schools must be reformed to cater to algebra requirements to be acquired in the later grades.

Another perspective in the literature about primary school pupils' algebraic thinking is functional thinking (Martinez & Brizuela, 2006; Smith, 2008; Warren & Cooper, 2008; Warren, Cooper, & Lamb, 2006). According to Smith (2008), function means "representational thinking that focuses on the relationship between two (or more) varying quantities" (p. 143). As such, early exposure to exploring the relationship of varying quantities and generalising about it will enable students to have a better understanding of functions while learning formal algebra (Warren & Cooper, 2008). Activities such as looking for patterns and generalising those patterns would provide a strong basis for developing functional thinking at primary school level. Many studies have revealed that young children are capable of thinking functionally (Lannin et al., 2006; Martinez & Brizuela, 2006). Evidence from past studies showed that children are able to see the relationship between input and output values (Stacey, 1989); generate the 'rule' (Lannin et al., 2006); use the 'rule' to find subsequent pattern (Carraher, Martinez, & Schliemann, 2008); and are able to represent the 'rule' verbally or symbolically (Carraher et al., 2008; Warren & Cooper, 2008).

In addition, primary school pupils' algebraic thinking also focuses on the variable. Although primary school pupils may be unable to understand the true meaning of a variable, they are able to use variables to represent unknowns and to represent varying quantities (Blanton et al., 2015; Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005; Mestre & Oliveira, 2012). The misconceptions posed by middle and high school students about variables are the major stumbling blocks in learning algebra (McNeil et al., 2010). To ascertain this, McNeil and colleagues (2010), carried out a study using 322 middle school students (sixth to eighth grade) assigned to three different conditions in which they were required to explain the mnemonic and non-mnemonic symbols (i.e., *c* and *b*; *x* and *y*; Φ and Ψ) used in an expression. Whereby, in the problem context the symbols represent the price of cake and brownie. The findings showed that students

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in the *c* and *b* condition misinterpreted the symbols mostly by giving explanation that *c* and *b* stand for the first letter of cake and brownie respectively.

To overcome this shortcoming, Knuth et al. (2005) suggested that teaching materials which encourage viewing literal symbol as variable could minimise student misconceptions about variables. Problem contexts which can evoke students to represent unknowns with variable representations can be encouraged at primary school level as suggested by Blanton et al. (2015). The authors' intervention has proven that grade three students taught by such teaching materials could represent unknown values by symbol notation and were able to relate the notations with the problem contexts.

As discussed in preceding sections, the strands of modelling, generalised arithmetic, and function play an important role in measuring primary school pupils' algebraic thinking. A scale which consists of items from all these three strands would give a clear picture of the primary school pupils' algebraic thinking. These indicators collectively explain the children's ability in thinking algebraically. These three strands of algebraic thinking explain the direction of the indicators to the algebraic thinking construct.

It is clear that the three strands may not share common traits in order to be reflected as algebraic thinking. The definitions of modelling, generalised arithmetic, and function carry different perspectives. Therefore, they do not have common traits about the construct. Hence, it would be inappropriate to interchange one with another as it may not be represented under one common concept. As such, excluding any one item or strand might affect the conceptual characterisation of algebraic thinking. These characteristics are the main concerns of representing algebraic thinking as reflective measurement models. To be more precise, these three strands are not correlated to each other. With regard to this, high correlation among them is not necessary as required by reflective measurement models. In addition, the three strands are independent and capture specific aspects. In sum, the strands of modelling, generalised arithmetic, and function are not interchangeable. Therefore, most likely they have different results. In fact, that is how it should be because they should not be correlated with each other. Therefore, the algebraic thinking latent constructs are best viewed through a formative measurement model. In other words, algebraic thinking strands can be represented as a hierarchical component model with the three strands being the lower-order components. Besides this, each strand has some formative indicators which measure different aspects. Hence, algebraic thinking has been modelled as a hierarchical component model of the formative-formative type in this study.

MATERIALS AND METHODS

Sample and data collection procedure

Based on the rule of thumb, the minimum sample size would be "10 times the largest number of formative indicators used to measure a single construct" (Hair et al., 214, p. 20). However, the sample size table created by Krejcie and Morgan (1970) was used to determine the sample size because the national schools' year five population was finite. A complete list of national schools in a particular district of Malacca was obtained from the Ministry of Education. The total number of students in the particular district of Malacca is 5347. According to Krejcie and Morgan (1970), sample size should be 357 for a population size of 5000. However, the researcher took a bigger sample of 539 to ensure yielding more precise item weights and loadings. A bigger sample size also provided better validity evidence. The Algebraic Thinking Diagnostic Assessment was administered to a total of 539 year five pupils in the particular district in Malacca. They were 275 (51%) males and 264 (49%) females. Only national schools were involved in this study. The schools were selected randomly. The researcher used *Rand()* function in Microsoft Excel 2013 to generate random numbers to be associated with each school in the complete list received from the Ministry of Education. The schools were then arranged in ascending order according to the random numbers generated. The first 10 schools were selected. Two or three classes were chosen based on each school principal's recommendation on their ability to solve the mathematical items.

Instruments

The present study adapted the algebraic thinking diagnostic assessment (ATDA) which was originally developed by Ralston (2013). ATDA comprised 28 items to assess the strands of modelling (10 items), generalised arithmetic (10 items), and function (8 items). Some 26 items were marked dichotomously; 1 for a correct answer while 0 for an incorrect answer. Two items (i.e., Q12exp and Q15exp) which demand explanations were marked according to the rubrics created by Ralston (2013) ranging from 0 to 2.

Each item in the ATDA was presented in both English and Malay languages to avoid language factor from influencing the data. ATDA consists of three major strands; modelling, generalised arithmetic, and function. Formative measurement model of ATDA comprises two levels. Lower order component includes three sub-constructs underlying primary school pupils' algebraic thinking. Algebraic thinking construct acts as a higher-order component. The formative measurement model was chosen to evaluate ATDA validity as each indicator carries a different weight. For instance, items from function assess the ability to work with numerical or figural patterns, while generalised arithmetic assesses the ability to generalise based on properties of arithmetic. Since each construct is totally different, therefore each carries a different weight. Hence, it should not be grouped as reflective indicators even though all the items are aimed at assessing algebraic thinking.

Data analysis technique
A model contains two levels of components which can be either reflective or formative to explain a general concept is known as a higher-order model or hierarchical component model (HCM) (Becker, Klein, & Wetzels, 2012). Commonly, the types of relationship between lower-order and higher-order component are classified into four categories of HCM (Hair et al., 2014; Ringle, Sarstedt, & Straub, 2012). These four categories of HCM are classified based on their relationship between higher-order component (HOC) and lower-order component (LOC) and respective indicators. Firstly, reflective-reflective type characterised as such based on the reflective relationship between HOC and LOC. In this type, reflective indicators measure each construct. The second type is known as reflective-formative, whereby higher-order components are formed by a common concept of a few reflective lower-order components. Thirdly, formative-reflective type indicates reflective relationships between LOC and HOC with each construct measured by formative indicators. Lastly, formative-formative type points out the formative relationship between LOC and HOC whereby each construct was measured by its formative indicators.

With regard to the aim of this study, the formative-formative type model has been employed in order to validate the items for the hierarchical component model of algebraic thinking. This model is depicted as shown in Figure 1. SmartPLS 3.2.4 software was used to carry out validation process of ATDA by conducting partial least squares (PLS) analysis. A few reasons were considered for choosing PLS data analysis technique.

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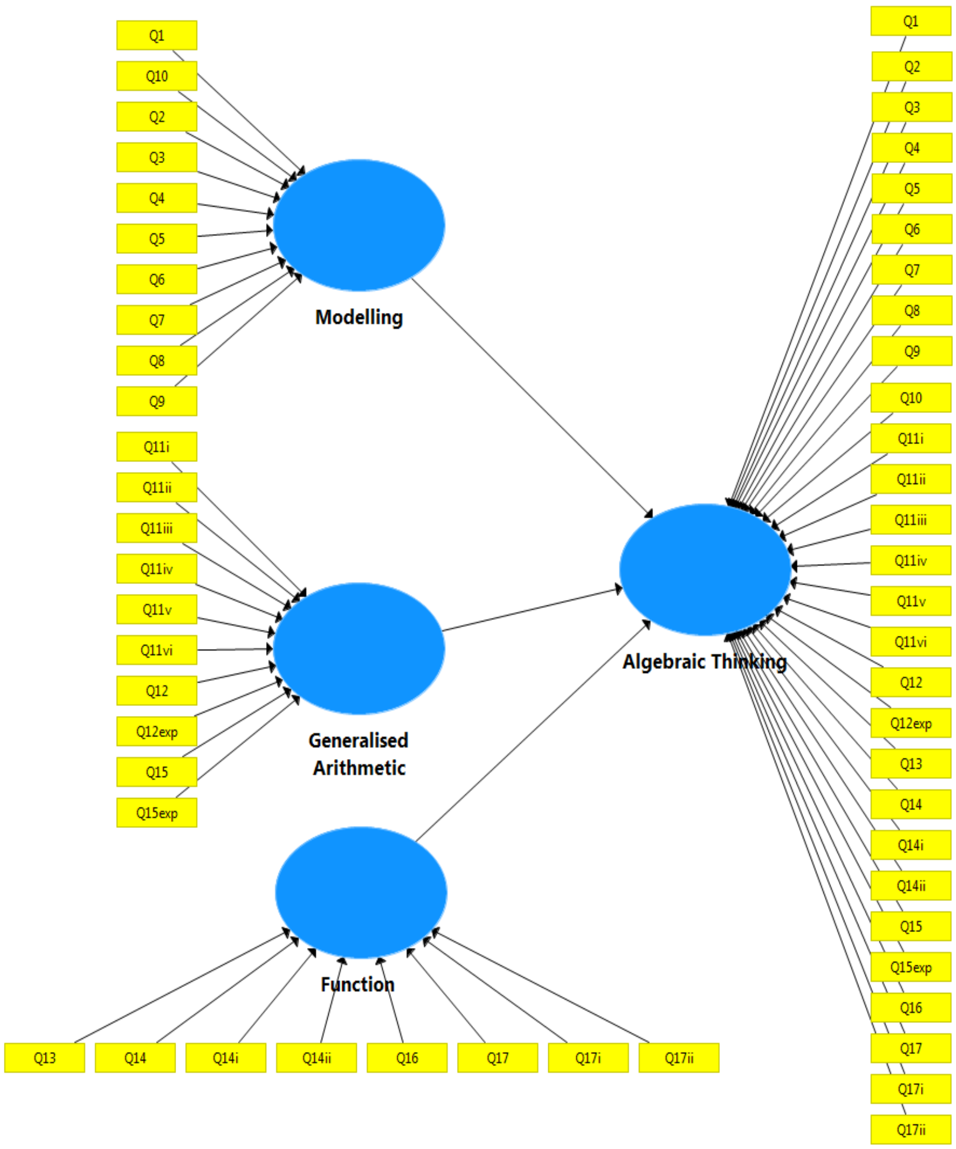

Figure 1. The two-stage approach

First and foremost, estimating the formative latent construct is easier using PLS-SEM approach (Hair et al., 2014; Henseler, Ringle, & Sinkovics, 2009). Then, measurement scale in PLS applies minimal requirement and sample size required for analysis is smaller compared to covariance-based structural equation modelling (CB-SEM) (Henseler et al., 2009). Other than this, the measurement model and structural model can be evaluated simultaneously in PLS-SEM which can avoid multicollinearity issues (Chin, 1998). Henseler et al. (2009) stated that "PLS path modelling is recommended in an early stage of theoretical development in order to test and validate exploratory models" (p. 282). To date there is no models have been evaluated in the field of primary pupils' algebraic thinking. This shows PLS-SEM is a predictive technique that can cater the purpose of the current study.

Respective item measures for latent constructs of modelling, generalised arithmetic, and function are shown in Table 1. In addition, these three strands of algebraic thinking do not have a common point which holds together the conceptual domain primary school pupils' algebraic thinking. Their respective item measures assess primary school pupils' ability in terms of modelling, arithmetic generalisation, and function. Hence, the relationships between algebraic thinking and its three strands act as higher and lower order components which have been manifested as second-order formative-formative type model as shown in Figure 1. With regard to this, the present study characterised algebraic thinking as a hierarchical component model and used the formative-formative type model. As mentioned earlier, PLS-SEM allows evaluation of measurement model and structural model simultaneously, thus the repeated indicator approach was used to avoid interpretational confusion (Becker, 2012). Especially, the three strands namely modelling, generalised arithmetic, and function are referred as first (lower) order components. A two-stage approach is utilised by representing algebraic thinking as second (higher) order component which was directly evaluated by the respective lower-order components and in return these three lower-order components were directly measured by their associated formative indicators.

Table 1
ATDA strands and its items

Strands	Items
1: Modelling	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10
2: Generalised arithmetic	Q11i, Q11ii, Q11iii, Q11iv, Q11v, Q11vi, Q12, Q12exp, Q15, Q15exp
3: Function	Q13, Q14, Q14i, Q14ii, Q16, Q17, Q17i, Q17ii

The relationship between modelling, generalised arithmetic, and function (LOC) with its formative indicators evaluated at lower-order components level. Followed by this, the relationship between the LOC and HOC (algebraic thinking) was determined at higher-order components level. In the present study, PLS analysis was used to estimate the measures in both LOC and HOC with a path weight scheme (Hair et al., 2014). In addition, standard error of the estimates was determined by bootstrapping with 5000 replications.

RESULTS AND DISCUSSION

According to Hair et al. (2014), normal distribution of data is not required in PLS-SEM as it is a non-parametric approach. Furthermore, when the sample size of 200 or more the non-normality impacts are negligible (Hair, Black, Babin, & Anderson, 2009). Since the present study used a sample size of 539, the non-normality is definitely not a concern for data analysis. Apart from this, the majority of the items scored dichotomously. Hence, test for normality would not be appropriate. The following sections will discuss common method variance, lower and higher components model assessments.

Common method variance

In accordance to Podsakoff, MacKenzie, Lee, and Podsakoff (2003), Harman's single factor test was carried out to assess common method bias. Based on the rule of thumb, the variance explained by single factor should be less than 50%. In this case, the maximum variance explained by a single factor is 19.62% only. Thus, it can be concluded that this dataset is free from common method bias.

Lower-order components model assessment

Three lower-order constructs are modelling, generalised arithmetic, and function which are measured by formative indicators. There are three criteria to evaluate formative measurement models (Hair et al., 2014). Firstly, assessing convergent validity of formative measurement model. This validity is not as meaningful as in reflective measurement model (Chin, 1998). Content validity would be more appropriate to ensure all major facets captured in the formative measurement model (Hair et al., 2014). Therefore this instrument was validated by a few field experts.

Secondly, multicollinearity issue should be examined. This is because highly correlated formative indicators might impact the weight estimations and their statistical significance. Besides that, high collinearity also can cause weights to be incorrectly estimated. A measure of collinearity is the variance inflation factor (VIF). There will be a collinearity problem if the VIF values do not fall within the range of 0.2 and 5 (Hair et al., 2014). In accordance to these criteria, Table 2 presents the values of VIF for each formative indicators involved in this study. As the VIF values for all three constructs are between 0.2 and 5, it can be concluded that collinearity is not an issue for the lower-order components of this model.

Thirdly, the relevance and significance of the formative indicators should be examined. It is assessed by examining the contribution of each formative indicator to the formative construct. This contribution is determined by outer weights and *t* values produced in bootstrapping to determine its significance. According to Hair et al. (2014), "when a formative indicator has non-significant outer weight but its outer loading is above 0.5, the indicator should be interpreted as

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absolutely important but not as relatively important." (p. 129). In addition, an indicator should be removed only when both outer weight and loading are insignificant as lack of empirical support.

Table 2
Collinearity statistics

Modelling		**Generalised arithmetic**		**Function**	
Indicators	VIF	Indicators	VIF	Indicators	VIF
Q1	1.225	Q11i	1.158	Q13	1.266
Q2	1.348	Q11ii	1.140	Q14	1.694
Q3	1.269	Q11iii	1.136	Q14i	1.834
Q4	1.797	Q11iv	1.290	Q14ii	1.289
Q5	2.425	Q11v	1.159	Q16	1.357
Q6	1.397	Q11vi	1.109	Q17	2.196
Q7	1.489	Q12	1.274	Q17i	2.278
Q8	1.756	Q12exp	1.493	Q17ii	1.419
Q9	2.494	Q15	1.175		
Q10	1.440	Q15exp	1.269		

Table 3 shows the results of bootstrapping procedure generating 5000 subsamples from 539 cases. Modelling has ten formative indicators in total. Seven indicators were significant at 95% and 99% confidence interval. Outer weights of Q1, Q4, and Q6 are insignificant. However, the outer loadings for Q4 and Q6 are more than 0.5 (i.e., 0.615 and 0.510 respectively). Thus, these two items were retained. As for Q1, the *t* value of outer loading was examined. This item also was retained as the *t* value (8.297) was significant. Nine indicators are significant at the 99% and 90% confidence interval in the generalised arithmetic construct. Q11iii has neither significant outer weight nor outer loading above 0.50. It was retained as it has significant *t* value (3.111) for outer loading. As for function construct, all indicators were significant at the 90%, 95%, and 99% confidence interval except Q14. It still remained in the model as its *t* value (6.947) for outer loading is significant even though the outer loading was 0.436. Table 3 provides evidence for relevance and significance criteria of the lower-order component model in this study. It can be concluded that the second evaluation criteria have been fulfilled and no issues prevent from proceeding with the evaluation of the higher-order component model.

Higher-order component model assessment

This section discusses the assessment of higher-order component model which involve modelling, generalised arithmetic, and function as lower-order components contribute to a latent variable of algebraic thinking as a higher-order component. Evaluation of higher-order component began with examining the collinearity issue among the three constructs. The assessment process is the same as procedures discussed in the preceding section (for lower-order components model).

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Table 3
Outer weights significance testing results for validity

Formative constructs	**Formative indicators**	**Outer weights (Outer loadings)**	***t* value**
Modelling	Q1	0.015 (0.383)	0.314^{NS}
	Q2	0.169 (0.505)	3.651***
	Q3	0.160 (0.484)	3.192***
	Q4	0.059 (0.615)	0.987 ^{NS}
	Q5	0.378 (0.789)	5.507***
	Q6	0.077 (0.510)	0.668 ^{NS}
	Q7	0.116 (0.579)	2.209**
	Q8	0.136 (0.695)	2.371**
	Q9	0.193 (0.804)	2.683***
	Q10	0.261 (0.539)	5.055***
Generalised arithmetic	Q11i	0.225 (0.383)	3.990***
	Q11ii	0.212 (0.431)	3.944***
	Q11iii	-0.036 (0.199)	0.652 ^{NS}
	Q11iv	0.393 (0.570)	6.989***
	Q11v	0.212 (0.395)	3.579***
	Q11vi	0.172 (0.312)	2.970***
	Q12	0.197 (0.495)	3.181***
	Q12exp	0.352 (0.684)	5.115***
	Q15	0.108 (0.317)	1.792 *
	Q15exp	0.200 (0.479)	3.118***
Function	Q13	0.347 (0.577)	6.053***
	Q14	0.075 (0.436)	1.018 ^{NS}
	Q14i	0.225 (0.596)	2.968***
	Q14ii	0.247 (0.565)	4.119***
	Q16	0.288 (0.645)	4.271***
	Q17	-0.143 (0.279)	1.683 *
	Q17i	0.211 (0.441)	2.490**
	Q17ii	0.375 (0.680)	5.955***

Note. NS = not significant. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$

Table 4 illustrates the values of VIF for all three indicators of the construct of algebraic thinking. Since the values are in between the range of 0.2 and 5, it can be concluded that there is no collinearity issue across the indicators. In addition, Table 4 also shows the three indicators are highly significant (99% confidence interval). Therefore, all the three constructs are accepted as formative indicators for their algebraic thinking latent variable construct. To be more specific, function construct yield the most weight (0.463) compared to other two indicators. Modelling and generalised arithmetic carry moderately equal weights (0.371 and 0.344 respectively). The nomological validity affirmed from the evidence that the three strands form algebraic thinking as formative constructs.

Table 4
Higher-order component validity results

Higher-order component	**Lower-order components**	**Outer weights**	***t* value**	**VIF**
Algebraic thinking	Modelling	0.371	31.466***	2.077
	Generalised arithmetic	0.344	24.024***	1.703
	Function	0.463	32.190***	1.607

Note. $^{***}p < 0.01$

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CONCLUSION

To date, no measurement model validation has been done on primary school pupils' algebraic thinking instrument. The present study was designed to address this gap. Findings showed that the three strands identified by Kaput (2008) to measure primary school pupils' algebraic thinking can be represented as a hierarchical component model. The present study has shown how the indicators can be used to form lower and higher order components to represent algebraic thinking as a hierarchical component model. In fact, this quantitative perspective of validation has provided evidence for more precise psychometric outcomes compared to past studies which widely measured algebraic thinking qualitatively. The findings have provided an in-depth view on the validation carried out in terms of indicator weights, multicollinearity issues, and constructs and indicators representation in the form of a formative-formative type model.

The findings of the present study shed some light on primary school pupils' algebraic thinking. They have provided new insights into the strands of algebraic thinking in primary school. If early algebra is not the same as algebra early, then what is it? The hierarchical component model using formative-formative type model in this study is self-explanatory to answer this question. It shows how different strands play an important role in forming algebraic thinking. Overall, no items have been removed. Though three items (i.e., Q1, Q11iii, and Q14) outer weight were not significant and outer loading less than 0.5, those items were retained as the outer loadings are significant and also those are theoretically driven conceptualised items. The findings of the present study established satisfactory levels of quality for formative constructs, thus providing evidence of the instrument validity for measuring year five pupils' algebraic thinking.

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